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The combustion of a gas suspension of particles reacting in accordance with a heterogeneous mechanism was considered in [1-4]. It was assumed that the reaction rate at the surface of the individual particles does not depend on the thickness of the oxide film, or that a film does not form, i.e., the reaction products are gaseous. With the oxidation of many metals the oxide film formed inhibits the reaction, i.e., with its growth, the rate of the reaction decreases. The special characteristics of the process of the combustion of individual particles of metals arising as a result of the effect of the oxide film were considered in [5], in which it was shown that the dependence of the reaction rate on the thickness of the film has a considerable effect on the laws governing the combustion of individual particles. In the present work, a study was made of the process of the combustion of a gas suspension of particles of metals oxidizing in accordance with the so-called parabolic law (the reaction rate is inversely proportional to the thickness of the oxide film). The results are compared with the laws governing the combustion of a gas suspension of particles reacting in accordance with a purely heterogeneous mechanism in the absence of an oxide film.

### 1. Statement of Problem

We shall consider the combustion of a gas suspension in a vessel, assuming that the temperature of the gas is identical over the whole volume and that the heat losses from the vessel are proportional to the difference between the temperatures of the gas and the walls of the vessel.

Assuming that all the particles are of identical size and that they are uniformly distributed in the vessel, we write the equations of the heating of an individual particle, of the balance of the thermal energy of the gas, and of the rate of growth of the oxide film

$$\frac{d\theta}{d\tau} = \frac{1}{z^n} \exp \frac{\theta}{1+\beta\theta} - \frac{\theta - \theta_g}{\Omega} \quad (1.1)$$

$$\frac{d\theta_g}{d\tau} = \frac{B}{1-B} \frac{\theta - \theta_g}{\Omega} - \frac{B}{1-B} \frac{\theta_g - \theta_w}{A\Omega} \quad (1.2)$$

$$\frac{dz}{d\tau} = \frac{1}{z^n} \exp \frac{\theta}{1+\beta\theta} \quad (1.3)$$

with the initial conditions

$$\tau = 0, \quad \theta = \theta_0, \quad \theta_g = \theta_{g0}, \quad z = z_0$$

$$\theta = \frac{E}{RT_*^2} (T - T_*), \quad \theta_g = \frac{E}{RT_*^2} (T_g - T_*),$$

$$z = \frac{3\delta}{r} \frac{QE}{cRT_*^2}, \quad z_0 = z(\delta_0)$$

$$\tau = \left( \frac{3QE}{cRT_*^2} \right)^{n-1} t \frac{kc_0^m}{r^{n+1}} \exp \left( - \frac{E}{RT_*} \right), \quad B = \frac{4/3\pi r^3 c_p N}{c_g \rho_g + 4/3\pi r^3 c_p N}$$

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$$\Omega = \left( \frac{3QE}{cRT_*^2} \right)^{n+1} \frac{c\rho}{3\lambda_g} \frac{kc_0^m}{r^{n-1}} \exp\left(-\frac{E}{RT_*}\right), \quad \beta = \frac{RT_*}{E} \quad (1.4)$$

$$A = \frac{\lambda_g}{2r} \text{Nu} \frac{4\pi r^2 VV}{\alpha_w S} \approx \frac{\rho_g}{\rho} \frac{B}{1-B} \frac{\text{Nu}}{\text{Nu}_w} \left(\frac{L}{r}\right)^2$$

Here  $T$ ,  $T_g$ ,  $T_w$  are the temperatures of the particles, the gas, and the wall of the vessel, respectively;  $T_*$  is the scale temperature, whose selection is elucidated during the course of the solution;  $B$  is the mass concentration of the particles in the suspension;  $\delta_0$ ,  $\delta$  are the initial and instantaneous thickness of the oxide film;  $r$  is the characteristic dimension of a particle;  $\lambda_g$  is the coefficient of thermal conductivity of the gas;  $c$ ,  $c_g$  are the specific heat capacities of the particles and the gas;  $\rho$ ,  $\rho_g$  are the densities of the particles and the gas;  $Q$  is the heat effect of the reaction per gram of film of oxide, multiplied by the ratio of the densities of the oxide and the metal;  $\text{Nu}$ ,  $\text{Nu}_w$ , are the Nusselt numbers of the heat transfer between the particles and the gas and between the gas suspension and the walls of the vessel, respectively;  $E$  is the activation energy;  $k$  is the preexponent in the law of the rate of growth of the oxide film;  $c_0$  is the concentration of the oxidizer at the surface of a particle;  $m$  is the order of the reaction with respect to the oxidizer;  $\alpha_w$  is the heat-transfer coefficient from the gas suspension to the walls of the vessel,  $\alpha_w = \text{Nu}_w \lambda_g / L$ ;  $L$ ,  $V$  are the characteristic dimension and volume of the vessel;  $S$  is the heat-transfer surface of the vessel;  $N$  is the number of particles in unit volume of the gas suspension;  $t$  is the time;  $n$  is the exponent in the oxidation law ( $n = 1$  represents a parabolic oxidation law).

The first term in the right-hand part of Eq. (1.1) describes diffusional conditions of the oxidation of a metallic particle under which the rate of the reaction is limited by the diffusion of the oxidizer through the oxide film; the diffusion coefficient is assumed to depend on the temperature in accordance with the Arrhenius formula with the activation energy  $E$ . This expression enters also into the expression for the rate of growth of the oxide film. The remaining terms in the equations describe the heat transfer between the particles and the gas, and between the gas suspension and the walls of the vessel.

The system (1.1)-(1.3) is written in the approximation of a two-temperature continuous medium. This imposes a limitation on the mass concentration of the particles, since in this approximation the distance between the particles must be greater than the size of the particles (at pressures on the order of 1 atm  $B \leq 0.45-0.5$ ).

## 2. Combustion of a Thermally Insulated Suspension

Let us consider the case where the evolution of heat in the wall of the vessel may be neglected. As analysis shows, this can be done, for example, when the temperature of the walls  $T_w$  exceeds the critical combustion temperature due to heat losses from the whole gas suspension in some characteristic ranges of  $RT_w^2/E$ , so that a strong rise in the reaction rate takes place until intense heat transfer from the walls of the vessel sets in. From system (1.1)-(1.3) we obtain the balance of the total energy of the gas suspension in the form

$$z - z_0 = (\theta - \theta_0) + (1 - B) B^{-1} (\theta_g - \theta_{g0}) \quad (2.1)$$

The time is eliminated from the remaining two equations by the division of (1.1) by (1.3).

It is not possible to evaluate Eq. (2.2) in analytical form. Numerical calculations were made which made it possible to bring out a number of the special characteristics of the combustion process, and to find an approximation which made it possible to evaluate the combustion time analytically.

$$\frac{d\theta}{dz} = 1 - z \exp\left(-\frac{\theta}{1+\beta\theta}\right) \left[ \frac{\theta - Bz}{1-B} + \frac{B}{1-B} (z_0 - \theta_0) - \theta_{g^*} \right] / \Omega \quad (2.2)$$

In what follows, we shall neglect the small parameter  $z_0$  (in accordance with [5]  $z_0 \ll 1$ ). We shall first consider the case of identical initial temperatures of the particles and the gas, and we shall then correlate the results taking account of their difference.

Taking  $T_*$  equal to  $T_0$ , we obtain  $\theta_0 = \theta_{g0} = 0$ . In this case, the behavior of the integral curves of Eq. (2.2) depends on two parameters, i.e., the mass concentration of the particles  $B$  and the dimensionless rate of the chemical reaction  $\Omega$ .

It is expedient to connect the different types of integral curves and their corresponding combustion conditions with the relative disposition of the integral curves and the zero isocline of Eq. (2.2). Setting  $d\theta/dz = 0$  in Eq. (2.2) we obtain

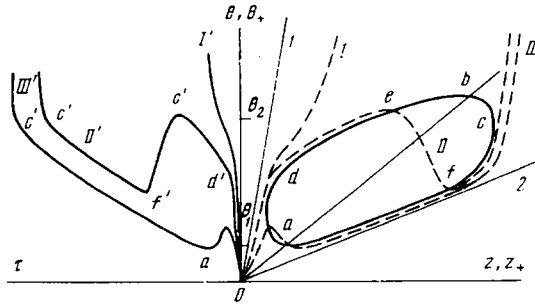


Fig. 1

$$z = \frac{1}{2B} \left[ \theta_+ \pm \sqrt{\theta_+^2 - 4u \exp \frac{\theta_+}{1 + 3\theta_+}} \right], \quad u = B(1 - B) \Omega \quad (2.3)$$

The values of the quantities at the isocline are designated by the "plus" sign for the difference from the corresponding values at the integral curve. A zero isocline exists with

$$u < u_0 = \exp \left( -\frac{2}{1 - 4\beta} \right) / (1 - 4\beta)^2 \quad (2.4)$$

and is a closed curve in whose internal region, for the integral curves  $d\theta/dz < 0$ , and in whose external region,  $d\theta/dz > 0$  (the isocline  $\theta_+(z_+)$  and the integral curves  $\theta(z)$  are plotted on the right-hand half of Fig. 1, and the dependence of the temperature of the particles on the time under different conditions is plotted on the left-hand half).

Straight lines 1 and 2 on Fig. 1 bound the region within which the integral curves (2.2) can lie. Straight line 1, bounding the region from above, corresponds to the equation  $\theta = z$  and to heating of the particles without heat transfer with the gas ( $\Omega \rightarrow \infty$ ). Straight line 2 corresponds to homogeneous heating of the whole gas suspension ( $\Omega \rightarrow 0$ ), with which  $\theta = \theta_g$ , and is described by the equation  $\theta = Bz$ ; it bounds the region from below.

Let us follow the behavior of the integral curves with different values of the parameters  $B$  and  $\Omega$ , and let us give a qualitative description of the corresponding combustion conditions. All the integral curves depart from the origin of coordinates with a positive derivative. At large values of the parameter  $\Omega$  with motion along the integral curve, the derivative  $d\theta/dz$  always remains positive (curve 1, I' on Fig. 1). This means that there is a monotonic rise in the temperature of the particles, i.e., the growth of the oxide film does not inhibit the development of the chemical reactions. These combustion conditions (conditions I) are analogous to the combustion conditions of isolated particles of metal.

With smaller values of the parameter  $\Omega$ , the derivative  $d\theta/dz$  decreases along the integral curve and with some value of  $\Omega$  reverts to zero (under these circumstances, the integral curve touches the zero isocline at the point  $b$  corresponding to a minimum of the curve of the isocline). This means that the growth of the oxide film and of the heat transfer from the particles to the gas start to inhibit the progressive development of the chemical reaction taking place as a result of heating of the particle. With a further decrease in the value of  $\Omega$ , the integral curve intersects the isocline (here, the less the value of  $\Omega$ , the further the points of intersection  $e, e'$  lie from the point  $b$ ) and falls into the region where  $d\theta/dz < 0$ . In this region, as a result of the growth of the oxide film, there is a decrease in the temperature of the particles until the integral curve intersects the lower branch of the zero isocline (the point  $f, f'$ ). After repeated intersection,  $d\theta/dz$  again becomes positive and, under these circumstances, the integral curve moves along the zero isocline right up to a temperature corresponding to the point  $c, c'$ . The type of integral curves described (II, II' on Fig. 1) corresponds to conditions II.

In the sections  $de, d'e'$  and  $fc, f'c'$ , the integral curve passes near the zero isocline, where  $d\theta/dz \approx 1$ . This means that the self-heating of the gas suspension takes place in a quasi-steady-state manner: at every moment there is thermal equilibrium between the particles and the gas, shifting slowly as a result of the accumulation of heat in the gas and the growth of the oxide film. In what follows, the stages  $de$  and  $fc$  will be called the high-temperature quasi-steady-state and the low-temperature quasi-steady-state stages, respectively.

In the high-temperature quasi-steady-state stage, the oxidation rate and the drop between the temperatures of the particles and the gas are great in comparison with the analogous characteristics in the low-temperature quasi-steady-state stage (the line  $\theta_g(z)$  always lies below the straight line 2 on Fig. 1). During the course of preliminary heating, conditions are attained in the low-temperature quasi-steady-state stage (the temperature  $\theta_2$ , point c, Fig. 1) under which thermal equilibrium becomes impossible and there is ignition of the gas suspension, i.e., a sharp breakaway of the temperature of the particles from the temperature of the gas.

With a still greater decrease in the parameter  $\Omega$ , the point of intersection of the integral curve by the zero isocline is shifted toward the point  $a$  of the zero isocline, and the integral curve passes along the whole lower half of the zero isocline. This case corresponds to strong inhibition of the chemical reaction by the oxide film.

Under these conditions, the low-temperature quasi-steady-state stages (segments  $ac$ ,  $a'c'$ ) are preceded by an unsteady-state oxidation process, with passage of the temperature of the particles through a maximum and a minimum. Under these conditions (condition III), the principal contribution to the induction period is made by the low-temperature quasi-steady-state stages. Numerical solution of the complete system (1.1)–(1.3) shows that the gas suspension arrives at the low-temperature quasi-steady-state stage at a temperature close to  $\theta_1$  (point  $a$  on Fig. 1).

The preliminary heating, with the combustion of a gas suspension of particles reacting in accordance with a heterogeneous mechanism ( $n = 0$ ), may also take place in a quasi-steady-state manner [3]; the difference consists in the fact that the shift in the equilibrium takes place only as a result of the accumulation of heat in the gas, and the low-temperature quasi-steady-state stage is attained without passage of the temperature of the particles through a maximum and a minimum.

The boundaries separating conditions I, II, III were calculated numerically (lines 1, 2 on Fig. 2). Conditions I are realized in the region of parameters above line 1, and conditions III in the region below line 2; conditions II correspond to the narrow region of parameters between lines 1 and 2. The isocline exists in the region of parameters below line 3 [see Eq. (2.4)]. Under conditions I, with sufficiently large values of the parameter  $\Omega$  (above line 4), each particle burns under almost adiabatic conditions.

The difference between conditions I, II, III with respect to induction periods and temperatures is substantial only in the case where the isocline occupies a large interval with respect to  $\theta$  and  $z$ . It is shown below that the dimensions of the isocline are determined by the degree of removal from the limit of the existence of the isocline (line 3 on Fig. 2). With a rise in the concentration of the particles, the boundaries 1 and 2 on Fig. 2 approach the limit 3 and the difference between the conditions vanishes.

The quasi-steady-state nature of the preliminary heating under conditions III offers the possibility of obtaining an approximate analytical expression for the induction period.

Let us make further calculations using the Frank–Kamenetskii expansion [6] of  $\beta = 0$ . Let us evaluate the value of the quasi-steady-state interval  $[\theta_1, \theta_2]$ . Calculating  $d\theta_+/dz_+$  from Eq. (2.3), and setting  $d\theta_+/dz_+ = 0$  with  $\theta = \theta_1$ ,  $d\theta_+/dz_+ = \infty$  with  $\theta = \theta_2$ , we obtain the following relationships determining  $\theta_1$  and  $\theta_2$

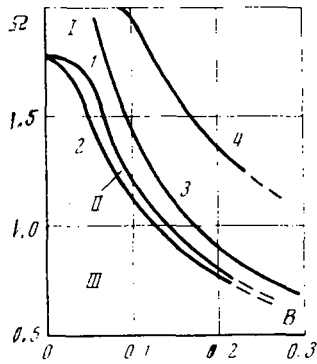
$$\ln[(\theta_2 - 1)/u] = \theta_2, \quad \ln(\theta_1^2/4u) = \theta_1 \quad (2.5)$$

A graphical solution of both equations is illustrated on Fig. 3. With increasing distance from limit of the existence of the isocline ( $\ln u^{-1} = 2$ ), the quasi-steady-state temperature interval  $(\theta_2 - \theta_1)$  increases and, with approach to the limit, decreases as  $(\ln u^{-1} - 2)^{1/2}$ .

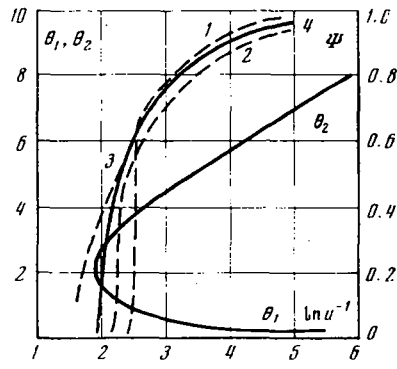
The temperatures  $\theta_1$  and  $\theta_2$  depend on the mass concentration of particles, in distinction from a gas suspension of particles reacting in accordance with a heterogeneous mechanism [3]. This is explained by the fact that the oxidation rate of the particles at a given temperature depends on the conditions of the heating of the particles up to this temperature.

To obtain an approximate formula for the induction period ( $\tau_i$ ), we integrate the system of Eqs. (1.1)–(1.3), setting  $d\theta/d\tau = 0$  in Eq. (1.1)

$$\tau_i = \frac{1}{B} \int_{\theta_1}^{\theta_2} z(\theta) \exp(-\theta) d\theta - \frac{\Omega(1-B)}{B} \int_{\theta_1}^{\theta_2} \left[ 1 - \frac{d \ln z}{d\theta} \right] d\theta \quad (2.6)$$



Eq. 2



Eq. 3

where  $z(\theta)$  is the equation of the integral curve in the plane  $\theta z$  (Fig. 1). Taking into account that in the low-temperature quasi-steady-state stage, the integral curve 1 is close to the isocline, we can substitute  $z(\theta)$ , determined from Eq. (2.3), into Eq. (2.6). The expression obtained in this case is simplified if the difference between the temperatures of the particles and the gas is neglected. This follows formally from Eq. (2.3) with  $\ln u^{-1} \gg 2$ , where the second term in the expression under the radical sign is small in comparison with the first, and Eq. (2.3) coincides approximately with the relationship (2.1), written for a homogeneous gas suspension  $z_+ = \theta_+/B$ . Integration of Eq. (2.6) gives

$$\tau_i = \left\{ [(1-\theta_1) \exp(-\theta_1) - (1+\theta_2) \exp(-\theta_2)] + u \left[ \ln \frac{\theta_2}{\theta_1} - (\theta_2 - \theta_1) \right] \right\} \frac{1}{B^2} = \frac{\Psi(\ln u^{-1})}{B^2} \quad (2.7)$$

The function  $\Psi(\ln u^{-1})$  is plotted on Fig. 3 (curve 4). A comparison of computer calculations of the induction period as a function of  $\Omega$  and  $B$  (the dotted lines 1, 2, 3 on Fig. 3 correspond to  $B = 0.5, 0.091, 0.17$ ) and calculations using formula (2.7) show that the approximate solution (2.7) is close to the exact solution under conditions III and differs sharply from it under conditions I and II.

The sharp change in the induction period in narrow intervals with respect to  $\Omega$  and  $B$  (numerical solution) is connected with the transition from conditions III to conditions I through conditions II.

In a gas suspension of particles reacting in accordance with a heterogeneous mechanism (this case is realized if  $n = 0$  in the starting system), conditions II do not exist, and the transition from conditions III to conditions I takes place smoothly without sharp changes, since, at the boundary separating conditions III and I, the value of the quasi-steady-state interval  $(\theta_2 - \theta_1)$  reverts to zero. In the case  $n = 1$ , the value of  $(\theta_2 - \theta_1)$  reverts to zero at the line 3 (Fig. 2), and the transition from conditions III to conditions I takes place in region II, which does not intersect line 3.

With an increase in the concentration of particles, the induction period in a gas suspension with  $n = 1$  is more sharply curtailed ( $\tau_i \sim B^{-2}$ ) than with  $n = 0$  ( $\tau_i \sim B^{-1}$  [3]). This difference is due to the fact that in the case  $n = 1$  the concentration of particles affects not only the total reactive surface of the particles, as  $n = 0$  [3], but also the thickness of the oxide film and the value of the quasi-steady-state temperature interval  $(\theta_2 - \theta_1)$ .

Formula (2.7) can be used to evaluate the effect of the melting point  $\theta_m$ , at which the oxide film loses its protective properties, on the induction period. If the temperature  $\theta_m$  falls into the interval  $[\theta_1, \theta_2]$ , then, in Eq. (2.7),  $\theta_2$  must be replaced by  $\theta_m$ .

Under these circumstances, the induction period is shortened and on the dependence  $\tau_i(\Omega, B)$  there appears a point of inflection corresponding to the equality  $\theta_2(\Omega, B) = \theta_m$ .

With the combustion of particles without the inhibiting effect of the oxide film, the breakaway from the low-temperature quasi-steady-state stage takes place at the ignition temperature of an isolated particle ( $T_i$ ). For particles of metals with an oxide film, this condition is not satisfied. To convince ourselves of this, we express the dimensionless ignition temperature of an isolated particle  $\theta_i = (E/RT_0^2)(T_i - T_0)$  in terms of  $\Omega$ . This value of the parameter  $\Omega$  corresponds to an initial temperature  $T_0 = ER^{-1} \ln \Omega^{-1} + \text{const}$ . It is shown in [5] that with  $\Omega = 1.57$ ,  $T_0$  is equal to the ignition temperature of a single particle  $T_i$ . Expressing  $(T_i - T_0)$  in terms of  $\Omega$ , we obtain  $\theta_i = (T_i/T_0)$  in  $(1.57/\Omega)$ . A comparison of the analytical expressions for  $\theta_1$  and  $\theta_2$  shows that  $\theta_1 < \theta_2$ .

By way of example, let us make a calculation using the thermophysical and kinetics constants from [7]

$$\begin{aligned}
c &= 0.214 \text{ cal/(g} \cdot \text{deg)} \quad \rho = 2.7 \text{ g/cm}^2 \\
\lambda_g &= 2.4 \cdot 10^{-4} \text{ cal/cm} \cdot \text{sec} \cdot \text{deg} \quad \dot{Q} = 4600 \text{ cal/g} \\
E &= 33000 \text{ cal/mole;} \\
kc_0^m \exp(-E/RT_0) &= 0.65 \cdot 10^{-3} \exp(-33000/RT_0) \text{ cm}^2/\text{sec}
\end{aligned}$$

The ignition temperature of an isolated particle, corresponding to these constants, is equal to 1300° K. If the initial temperature is 1000° K, then,  $\Omega \approx 0.04$ . With these values of  $\Omega$ , the gas suspension will burn under conditions III with an induction period (with  $B \approx 0.1$  and  $r = 4 \cdot 10^{-3}$  cm)  $t_i \approx 27$  sec, which is two orders of magnitude greater than the adiabatic induction period of an isolated particle.

Let us pass on to the case of different initial temperatures of the particles and the gas. With  $T_0 < T_{g0}$ , there exist the same three sets of combustion conditions. The stages discussed above are preceded by the process of equalization of the temperature of the particles and the gas; the equalization temperature is close to the temperature of inert mixing  $T_k = BT_0 + (1-B)T_{g0}$ . After the inert heating of the particles up to  $T_k$ , the gas suspension arrives at the low-temperature quasi-steady-state stage as a result of chemical reactions. Since, with  $T_0 < T_{g0}$ , the temperature  $T_k$  is less than  $T_0$ , the thickness of the oxide film at the moment of arrival at the low-temperature quasi-steady-state stage will be greater than with equal initial temperatures of the particles and the gas. As a result of this, the rate of heat evolution in the gas suspension decreases, the time required for attaining the temperature of breakaway from the low-temperature quasi-steady-state stage increases, and, in addition, the time required for heating the particles up to  $T_k$  must be added to the residence time in the low-temperature quasi-steady-state stage. As calculations show, with the development of the low-temperature quasi-steady-state stage, this time is small in comparison with the time required for quasi-steady-state development of the process and may be neglected.

Choosing the initial temperature of the gas as the scale temperature  $T_*$ , and determining  $z_+(\theta_+)$ ,  $\theta_1$ ,  $\theta_2$ ,  $\tau_i$  in accordance with the scheme expounded above, it can be shown that the induction period is expressed in terms of the function

$$\tau_i = \frac{1}{B^2} \exp\left(\frac{B}{1-B}|\theta_0|\right) \Psi\left[\ln u^{-1} + \frac{B}{1-B}|\theta_0|\right] \quad (2.8)$$

### 3. Effect of Heat Removal and Burning-Out

The combustion of a gas suspension of particles with an oxide film, as well as of particles reacting in accordance with a heterogeneous mechanism [2, 3], is limited by heat removal in the wall of the vessel and by burning-out of the particles.

With the presence of heat removal, it is impossible to obtain relationships (2.1), determining the total reserve of energy in the system. However, during the course of the process under conditions III, the temperatures of the particles and the gas in the low-temperature quasi-steady-state stage differ only slightly, which permits isolating a parameter determining the critical conditions in the region of the parameters where the ignition temperature of the suspension is less than the ignition temperature of an isolated particle. Introducing the new variables  $x = \tau[B(1-B)^{-1}]^2$ ,  $y = zB(1-B)^{-1}$ , and combining Eq. (1.1), multiplied by  $B(1-B)^{-1}$ , with Eq. (1.2), in place of (1.1)-(1.3) we obtain

$$\frac{d(\theta_g + B\theta(1-B))}{dx} = \frac{1}{y} \exp\left[\frac{\theta}{1+\beta\theta}\right] - \frac{1-B}{B} \frac{\theta_g - \theta_w}{A\Omega} \quad (3.1)$$

$$dy/dx = \frac{1}{y} \exp\left[\frac{\theta}{1+\beta\theta}\right] \quad (3.2)$$

$$x = 0, \quad \theta = \theta_g = 0, \quad y = Bz_0/(1-B) \ll 1 \quad (3.3)$$

The critical condition for ignition is the connection between the parameters of the problem (3.1)-(3.3)

$$A\Omega B / (1-B) = f[\beta, z_0 B / (1-B), B / (1-B)] \quad (3.4)$$

Neglecting the difference between the temperatures of the particles and the gas, we see that in place of the parameter  $B/(1-B)$  standing under the differentiation sign in Eq. (3.1), there appears the parameter  $1/(1-B)$ , which is insignificant with small concentrations. The small parameter  $z_0 B/(1-B)$  also has only a slight effect on the critical condition; therefore, with a fixed value of  $\beta$ , the right-hand part of relationship (3.4) is constant. A numerical solution of the complete system (1.1)-(1.4) with  $\beta = 0$  is in satisfactory agreement with the approximate result (3.4) if we set  $f = 1.2$ .

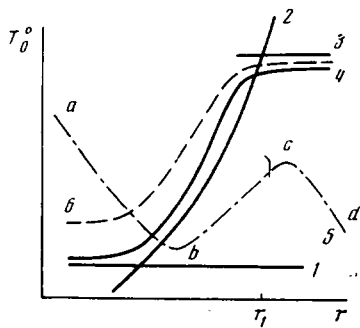


Fig. 4

The critical conditions with respect to heat removal for a gas suspension of particles reacting in accordance with a heterogeneous mechanism are bound up with the impossibility of a steady-state course of the reaction, while for a gas suspension of metallic particles, they are bound up with the existence of a breakaway temperature from the low-temperature quasi-steady-state stage (under critical conditions, the breakaway temperature  $\theta_2$  vanishes, in view of the breaking of the isocline).

Since

$$A \sim B / (1 - B) r^2, \quad \Omega \sim r^{1-n} \exp(-E / RT_0)$$

then the connection between the critical temperature  $T_0^o$  and the particle size is determined by the relationship

$$\exp(-E / RT_0^o) = \text{const } r^{n+1} (1 - B)^2 / B^2 \quad (3.5)$$

With a decrease in the particle size, the effect of a decrease in the reaction surface ( $n = 0$  [2]) is reinforced by the effect of inhibition of the chemical reaction by the oxide film ( $n = 1$ ); therefore,  $T_0^o$  with  $n = 1$  depends on  $r$  more strongly than in the case  $n = 0$  (on Fig. 4 the segment  $bc$  of the dotted line represents  $n = 0$ , and line 2 represents  $n = 1$ , using formula (3.5). Formula (3.4) is approximately applicable so long as the critical temperature of the suspension is less than the critical temperature for the ignition of an isolated particle (curve 2 on Fig. 4). With particle sizes greater than  $r_1$  ( $r_1$  is the particle size with which curves 2 and 3 intersect), the ignition temperature of the suspension is close to the ignition temperature of an isolated particle and does not depend on the particle size [5].

In the region of small particle sizes, the applicability of (3.4) is limited by the critical conditions for burning-out. The meaning of these conditions consists in the fact that the burning material in the gas suspension may not suffice to attain the temperature of a breakaway from low-temperature quasi-steady-state conditions, and the particles may be completely burned out in the low-temperature quasi-steady-state stage at a temperature close to the temperature of the gas. We write the critical conditions for burning-out in dimensional form

$$T_0^o = T_2 (\ln u^{-1}) - BQ / c \quad (3.6)$$

Relationship (3.6) is approximate since, in the starting system (1.1)-(1.3), no account was taken of burning-out of the oxidizer or of the change in the reaction surface of the particles during the course of the preliminary heating. The parameter  $\Omega \sim r^{1-n}$  with  $n = 1$  does not contain the particle size; therefore, the temperature of the breakaway from the low-temperature quasi-steady-state stage  $T_2 (\ln u^{-1})$  and, together with it, the critical initial temperature  $T_0^o$ , do not depend on the particle size (line 1 on Fig. 4). The complete dependence  $T_0^o(r)$  has an S-shaped form (Fig. 4, curve 4). With a decrease in the concentration of particles,  $T_0^o$  is shifted upward (curve 6 on Fig. 4).

For purposes of comparison, the dotted line 5 shows the dependence of the critical temperature on the particle size for a suspension of particles reacting in accordance with a heterogeneous mechanism ( $n = 0$  [2]). The sections of this curve  $ab$ ,  $bc$ ,  $cd$ , correspond to the critical conditions with respect to burning-out and heat removal, and to the critical condition for the ignition of an isolated particle.

The mutual disposition of curves 1 and 2 on Fig. 4 shows that a transition with respect to the particle size in a region where burning-out is considerable implies also a simultaneous increase in the degree of removal from the critical condition for heat removal, while in a region where combustion is limited by heat removal, burning-out can be neglected. This approximately justifies the separate derivation of the critical conditions for heat removal (3.4) and for burning-out (3.6).

#### LITERATURE CITED

1. L. A. Klyachko, "Combustion of a set of particles with a heterogeneous reaction," *Teploenergetika*, No. 8 (1966).
2. É. N. Rumanov and B. I. Khaikin, "Critical conditions for the auto-ignition of a set of particles," *Fiz. Goreniya i Vzryva*, 5, No. 1 (1969).
3. V. I. Lisitsyn, É. N. Rumanov, and B. I. Khaikin, "The induction period with the combustion of a set of particles," *Fiz. Goreniya i Vzryva*, 7, No. 1 (1971).

4. M. A. Gurevich, G. E. Ozerov, and A. M. Stepanov, "Critical conditions for the auto-ignition of a polydisperse gas suspension of a solid combustible," *Fiz. Goreniya i Vzryva*, 7, No. 1 (1971).
5. B. I. Khaikin, V. N. Bloshenko, and A. G. Merzhanov, "The combustion of metal particles," *Fiz. Goreniya i Vzryva*, 6, No. 4 (1970).
6. D. A. Frank-Kementskii, *Diffusion and Heat Transfer in Chemical Kinetics* [in Russian], Izd. Nauka, Moscow (1972).
7. P. F. Pokhil, A. F. Belyaev, Yu. V. Frolov, Yu. S. Logachev, and A. I. Korotkov, *Combustion of Powdered Metals in Active Media* [in Russian], Izd. Nauka, Moscow (1972).